Degrees of Freedom of Millimeter Wave Full-Duplex Systems With Partial CSIT

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***Abstract*—The degrees of freedom (DoF) of L-path *poor scattering* full-duplex (FD) systems is studied, in which a FD base station having M transmit antennas and M receive antennas supports a set of half-duplex mobile stations (MSs). Assuming no self-interference, a hybrid scheduling is proposed achieving the optimal sum DoF with *partial* channel state information at the transmitter side as the number of MSs increases. In particular, the proposed scheme combines a zero-forcing beamforming for uplink and a random transmit beamforming for downlink with opportunistic scheduling. It is shown that the optimal sum DoF of $2M$ is asymptotically achievable as long as the number of MSs scales faster than $\mathsf{snr}^{\min(M-1,L)+M}$, where snr denotes the signal-to-noise ratio.**

***Index Terms*—Degrees of freedom (DoF), full-duplex (FD), hybrid scheduling, inter-terminal interference, poor scattering.**

I. Introduction

FULL-DUPLEX (FD) technologies have recently been taken into account as a promising solution for boosting the spectral efficiency [1]. However, the potential advantage of FD systems may be limited by a new challenge—the inter-terminal interference—that does not appear in half-duplex (HD) systems. The problem of inter-terminal interference in FD systems has recently been studied in terms of degrees of freedom (DoF) [2], [3]. In particular, if channels are ergodic phase fading and full channel state information at the transmitter (CSIT) is available, then it was shown in [3] that the DoF of FD systems can be ideally twice as large as that of HD systems. Several inter-terminal interference cancellation schemes were shown in [2] for a 3-node FD system, but there are some practical challenges including the amount of CSI feedback bits.

In wireless communications systems, opportunistic transmission techniques have been widely studied for broadcast channels and interference channels [4], [5]. It was pointed that the same sum rate scaling law as the optimal dirty-paper coding can be achieved for broadcast channels via random beamforming with far less CSI feedback [4]. Moreover, using opportunistic communications, it is possible to asymptotically achieve one DoF per user under certain user scaling law conditions for multi-cell interference channels [5]. Note that for achieving these DoFs, the transmitters do not require the knowledge of the instantaneous channel realizations.

In practice, the potential benefits of multiple-antenna systems may be limited by rank deficiency of the channel matrix due to a poor scattering channel environment [6]. For example, because of the excessively high path loss and the sensitivity to blockages at millimeter wave (mmWave) frequency, mmWave channels are likely to have an inherent property of the poor scattering nature (see [7], [8] and references therein). Hence, it is essential to assess the effects of poor scattering on the DoF in FD systems. In this letter, we first introduce a *hybrid scheduling* into a FD system, which operates in an L-path *poor scattering* channel and is composed of a $2M$-antenna FD base station (BS) and a large number of single-antenna half-duplex mobile stations (MSs). Under the *partial CSIT* assumption, our hybrid scheme combines a random transmit beamforming for downlink along with a zero-forcing (ZF) filtering for uplink. As our main result, when M uplink and M downlink MSs are served through our scheduler, we show that the total DoF of $2M$ is achievable provided that the number of MSs scales faster than $\mathsf{snr}^{\min(M-1,L)+M}$, where snr denotes the signal-to-noise ratio (SNR). We remark that our scheme only requires each MS to feedback M real values along with the corresponding beamforming vector indices, which is significantly less than the full CSIT case.

II. System Model and Performance Metric

A. System Model

We consider a cellular system consisting of a FD BS having M transmit antennas and M receive antennas and a set of N HD MSs, each of which is equipped with a single antenna, where $N \geq 2M$. Each MS can be supported either as uplink or downlink, but not simultaneously. We assume that self-interference due to the FD operation at the BS is completely suppressed. Suppose that $\mathcal{N}^{(\mathrm{u})}$ and $\mathcal{N}^{(\mathrm{d})}$ are the sets of transmit and receive MSs, respectively, at a given time, satisfying that $\mathcal{N}^{(\mathrm{u})} \cap \mathcal{N}^{(\mathrm{d})} = \emptyset$ and $\mathcal{N}^{(\mathrm{u})} \cup \mathcal{N}^{(\mathrm{d})} = \{1, \cdots, N\}$. Then, the M-dimensional received signal vector at the BS (for uplink transmission) and the received signal at MS $j \in \mathcal{N}^{(\mathrm{d})}$ (for downlink transmission) are given by

$$\boldsymbol{y}^{(\mathrm{u})} = \sqrt{\mathsf{snr}} \sum_{i \in \mathcal{N}^{(\mathrm{u})}} \boldsymbol{h}_i^{(\mathrm{u})} x_i^{(\mathrm{u})} + \boldsymbol{z}^{(\mathrm{u})},$$
$$y_j^{(\mathrm{d})} = \sqrt{\mathsf{snr}}\, {\boldsymbol{h}_j^{(\mathrm{d})}}^{\dagger} \boldsymbol{x}^{(\mathrm{d})} + \sqrt{\mathsf{snr}} \sum_{i \in \mathcal{N}^{(\mathrm{u})}} g_{j,i} x_i^{(\mathrm{u})} + z_j^{(\mathrm{d})},$$

Manuscript received November 18, 2015; revised December 22, 2015; and accepted February 21, 2016. Date of publication XX XX, 2016; date of current version XX XX, 2016. This work was supported by ICT R&D program of MSIP/IITP. [B0101-15-1361, Development of PS-LTE System and Terminal for National Public Safety Service]. The associate editor coordinating the review of this paper and approving it for publication was Z. Wang.

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Digital Object Identifier XXX

respectively, where $\boldsymbol{h}_i^{(\mathrm{u})} \in \mathbb{C}^{M\times 1}$, $\boldsymbol{h}_j^{(\mathrm{d})} \in \mathbb{C}^{M\times 1}$, $g_{i,j} \in \mathbb{C}$ are the channels from MS i to the BS, from the BS to MS j, and from MS j to MS i, respectively (which will be specified by reflecting poor scattering characteristics later). The transmit signal of MS i and the transmit signal vector of the BS, denoted by $x_i^{(\mathrm{u})} \in \mathbb{C}$ and $\boldsymbol{x}^{(\mathrm{d})} \in \mathbb{C}^{M\times 1}$, should satisfy the average power constraint P, i.e., $\mathbb{E}\left\{\left|x_i^{(\mathrm{u})}\right|^2\right\} = 1$ and $\mathbb{E}\left\{\left\|\boldsymbol{x}^{(\mathrm{d})}\right\|^2\right\} = M$.[1] The additive noise vector at the BS and the additive noise at MS j, denoted by $\boldsymbol{z}^{(\mathrm{u})} \in \mathbb{C}^{M\times 1}$ and $z_j^{(\mathrm{d})} \in \mathbb{C}$, are assumed to follow $\mathcal{CN}(\mathbf{0},\mathbf{I})$ and $\mathcal{CN}(0,1)$, respectively.[2]

We adopt an L-scatterer geometric channel model [6], in which each scatterer is assumed to contribute a single propagation path between a transmitter and a receiver. Under this model, the downlink channel vector $\boldsymbol{h}_j^{(\mathrm{d})}$ ($j \in \mathcal{N}^{(\mathrm{d})}$) is expressed as

$$\boldsymbol{h}_j^{(\mathrm{d})}=\sqrt{\frac{1}{L}}\sum_{\ell=1}^{L}\alpha_j(\ell)\left[e^{\jmath\theta_j^{\ell}(1)}\cdots e^{\jmath\theta_j^{\ell}(M)}\right]^T=\sqrt{\frac{1}{L}}\boldsymbol{E}_j\boldsymbol{\alpha}_j, \quad (1)$$

where $\boldsymbol{e}_j^{\ell} = \left[e^{\jmath\theta_j^{\ell}(1)}\cdots e^{\jmath\theta_j^{\ell}(M)}\right]^T$ is the phase array of the lth scatterer; $\boldsymbol{E}_j = \left[\boldsymbol{e}_j^1 \cdots \boldsymbol{e}_j^L\right]$; $\alpha_j(\ell)$ is the amplitude of the ℓth scatterer; and $\boldsymbol{\alpha}_j = \left[\alpha_j(1)\cdots\alpha_j(L)\right]^T$. We assume that $\alpha_j(\ell)$ follows $\mathcal{CN}(0,1)$ and is independent of different j and ℓ. We further assume random phases, i.e., $\theta_j^{\ell}(m)$ is drawn uniformly at random over $[0,2\pi)$ and is independent of different j, ℓ, and m. In the same manner, the uplink channel vector $\boldsymbol{h}_i^{(\mathrm{u})}$ ($i \in \mathcal{N}^{(\mathrm{u})}$) can be expressed as $\boldsymbol{h}_j^{(\mathrm{u})} = \sqrt{\frac{1}{L}}\bar{\boldsymbol{E}}_j\bar{\boldsymbol{\alpha}}_j$, where $\bar{\boldsymbol{E}}_j$ is the random phase matrix and $\bar{\boldsymbol{\alpha}}_j$ is the amplitude vector. Lastly, $g_{j,i}$ is expressed as

$$g_{j,i} = \sqrt{\frac{1}{L}}\sum_{\ell=1}^{L}\beta_{j,i}(\ell)\,e^{\jmath\varphi_j^i(\ell)} = \sqrt{\frac{1}{L}}\boldsymbol{\psi}_{j,i}\boldsymbol{\beta}_{j,i}, \quad (2)$$

where $\boldsymbol{\psi}_{j,i} = \left[e^{\jmath\varphi_j^i(1)}\,e^{\jmath\varphi_j^i(2)}\cdots e^{\jmath\varphi_j^i(L)}\right]$ is the random phase vector from MS i and $\boldsymbol{\beta}_{j,i} = \left[\beta_{j,i}(1)\ \beta_{j,i}(2)\cdots\beta_{j,i}(L)\right]^T$ is the amplitude vector from MS i. We assume the block fading, i.e., once each channel is realized, it remains fixed during each coding or communication block. We further assume that full CSI is available at the receiver side, but only partial CSI is available at the transmitter side, which will be specified later.

B. Performance Metric

For an uplink and downlink sum-rate pair $(\mathsf{R}^{(\mathrm{u})}, \mathsf{R}^{(\mathrm{d})})$, the following sum DoF is achievable:

$$\mathsf{DoF} = \lim_{\mathsf{snr}\to\infty}\frac{\mathsf{R}^{(\mathrm{u})}+\mathsf{R}^{(\mathrm{d})}}{\log(\mathsf{snr})}. \quad (3)$$

The primary aim of this letter is to establish a simple scheduling and beamforming strategy achieving the optimal sum DoF of the L-path FD system under the partial CSIT assumption when the number of MSs is large enough.

III. Hybrid Scheduling and DoF Analysis

Our hybrid scheme employs different beamforming strategies for uplink and downlink. For uplink transmission, a zero-forcing (ZF) beamforming can be applied at the BS to null out uplink interference without CSIT. For downlink transmission, on the other hand, ZF at the BS is impossible without CSIT and moreover there exists MS-to-MS interference due to the FD operation.[3] To manage both downlink interference and MS-to-MS interference, for downlink transmission, we introduce an opportunistic scheduling combined with a random transmit beamforming. The overall procedure is described according to the following steps:

1. A set of M uplink MSs $\mathcal{S}^{(\mathrm{u})} = \{\phi_1,\cdots,\phi_M\}$ is chosen from $\{1,\cdots,N\}$ in a round-robin fashion, where the fairness among uplink MSs is automatically guaranteed, and each MS in $\mathcal{S}^{(\mathrm{u})}$ transmits its uplink packets.
2. The BS decodes uplink packets transmitted from $\mathcal{S}^{(\mathrm{u})}$ using the ZF receive beamforming.
3. The BS broadcasts M orthogonal beamforming vectors $\{\boldsymbol{v}_m \in \mathbb{C}^{M\times 1}\}_{m=1}^M$ to the MSs in $\{1,\cdots,N\}\setminus\mathcal{S}^{(\mathrm{u})}$, where $\{\boldsymbol{v}_m\}_{m=1}^M$ are generated according to an isotropic distribution.
4. Each MS $j \in \{1,\cdots,N\}\setminus\mathcal{S}^{(\mathrm{u})}$ computes
$$\mathcal{I}_{j,m} = \sum_{\ell=1,\ell\neq m}^{M}\left|\boldsymbol{h}_j^{(\mathrm{d})\dagger}\boldsymbol{v}_\ell\right|^2 + \sum_{\ell=1}^{M}\left|g_{j,\phi_\ell}\right|^2 \quad (4)$$
for all $m \in \{1,\cdots,M\}$ and feeds back $\{\mathcal{I}_{j,m}\}_{m=1}^M$ to the BS, where the first and second terms in (4) indicate the downlink and MS-to-MS interference powers, respectively.
5. For $m \in \{1,\cdots,M\}$, the BS selects
$$\pi_m = \operatorname*{arg\,min}_{j\in\{1,\cdots,N\}\setminus\left(\mathcal{S}^{(\mathrm{u})}\cup\{\pi_\ell\}_{\ell=1}^{m-1}\right)}\mathcal{I}_{j,m},$$
and constructs $\mathcal{S}^{(\mathrm{d})} = \{\pi_1,\cdots,\pi_M\}$, where $\{\pi_k\}_{k=1}^0 = \emptyset$. Then, the BS transmits its downlink packets to the MSs in $\mathcal{S}^{(\mathrm{d})}$ using $\{\boldsymbol{v}_m\}_{m=1}^M$, i.e., MS π_m is served by the beamforming vector $\boldsymbol{v}_m$.
6. Each MS in $\mathcal{S}^{(\mathrm{d})}$ decodes its downlink packets.

For the proposed scheme, we assume that each MS $j \in \{1,\cdots,N\}\setminus\mathcal{S}^{(\mathrm{u})}$ can track the MS-to-MS interference by overhearing uplink pilot signals, and the BS can attain $\{\mathcal{I}_{j,\ell}\}_{\ell=1}^M$ of each MS $j \in \{1,\cdots,N\}\setminus\mathcal{S}^{(\mathrm{u})}$ by feedback.

The following theorem shows that the propose scheme asymptotically achieves the optimal sum DoF as the number of MSs is greater than a certain level.

Theorem 1: For the L-path FD system in Section II, $\mathsf{DoF} = 2M$ is achievable with high probability (whp) if $N = \omega\left(\mathsf{snr}^{\beta}\right)$, where $\beta = \min(M-1, L) + M$.[4] $\diamond$

Note that this DoF achievability matches with the upper bound on the DoF obtained by assuming no MS-to-MS interference in the FD system as an ideal case. It is worthwhile to

[1] For easy presentation, we assume the average transmit power M for $\boldsymbol{x}^{(\mathrm{d})}$, but Theorem 1 still holds under the assumption of $\mathbb{E}\left\{\left\|\boldsymbol{x}^{(\mathrm{d})}\right\|^2\right\} = 1$.

[2] The notation $\mathbb{C}$ denotes the complex numbers; $(\cdot)^T$ and $(\cdot)^\dagger$ denote the transpose and conjugate transpose, respectively; $\mathbf{I}$ denotes the identity matrix; $\mathbb{E}\{\cdot\}$ denotes the expectation operator; and $\mathcal{CN}(\mathbf{m},\mathbf{\Sigma})$ denotes the complex Gaussian vector with mean vector $\mathbf{m}$ and covariance matrix $\mathbf{\Sigma}$.

[3] Unlike the strategy in [9] that applies a random receive beamforming at the BS, we apply the ZF receive beamforming, which can be performed without CSIT.

[4] We use the Bachmann–Landau notation: $f(x) = \mathcal{O}(g(x))$ if $\lim_{x\to x_0}\frac{f(x)}{g(x)} = c < \infty$ and $f(x) = \omega(g(x))$ if $\lim_{x\to x_0}\frac{f(x)}{g(x)} = \infty$.

mention that as L decreases, the number of required MSs to attain $\mathsf{DoF} = 2M$ decreases as proved in Theorem 1, which will also be demonstrated via numerical evaluation in Section IV. For the rest of this section, we prove Theorem 1. We assume that $\mathcal{S}^{(\mathrm{u})} = \{1, \cdots, M\}$ for notational convenience. It is obvious that the sum DoF of M is achievable for uplink transmission by ZF at the BS. Let us now focus on downlink transmission. For $m \in \{1, \cdots, M\}$, the received signal-to-interference-plus-noise ratio (SINR) at MS π_m is given by

$$\mathsf{sinr}_{\pi_m}^{(\mathrm{d})} = \frac{\left|\boldsymbol{h}_{\pi_m}^{(\mathrm{d})\dagger} \boldsymbol{v}_m\right|^2 \mathsf{snr}}{1 + \mathcal{I}_{\pi_m,m}\mathsf{snr}}. \quad (5)$$

Then, the downlink sum rate is given by

$$\mathsf{R}^{(\mathrm{d})} = \sum_{m=1}^{M} \log_2\left(1 + \mathsf{sinr}_{\pi_m}^{(\mathrm{d})}\right). \quad (6)$$

We first introduce the following important lemmas characterizing the asymptotic behavior of interference power.

Lemma 1 (Interference Power Statistics): Let $F_{\mathcal{I}_{j,m}}(\xi)$ denote the cumulative distribution function (CDF) of $\mathcal{I}_{j,m}$ in (4). Then,

$$F_{\mathcal{I}_{j,m}}(\xi) \geq \frac{e^{-1}\left(2\left(M-1\right)M\right)^{-\beta}}{\Gamma\left(\beta+1\right)}\xi^{\beta} \quad (7)$$

for $0 < \xi < 2M(M-1)$, where $\Gamma(\cdot)$ is the Euler gamma function.[5]

Proof: See Appendix A. □

Lemma 2 (Asymptotic Interference Power): For any constant $\epsilon > 0$ independent of snr, the probability that $\mathcal{I}_{\pi_m,m} \leq \frac{\epsilon}{\mathsf{snr}}$ for all $m \in \{1, \cdots, M\}$ is lower-bounded by

$$1 - M\left(1 - \frac{e^{-1}\left(2\left(M-1\right)M\right)^{-\beta}}{\Gamma\left(\beta+1\right)}\left(\frac{\epsilon}{\mathsf{snr}}\right)^{\beta}\right)^{N-2M+1}. \quad (8)$$

Proof: Let $\mathcal{A}_m \triangleq \{1, \cdots, N\} \setminus \left(\mathcal{S}^{(\mathrm{u})} \cup \{\pi_\ell\}_{\ell=1}^{m-1}\right)$ and $|\mathcal{A}_m|$ be the candidate set associated with the mth beamforming vector and its cardinality, respectively. Then for a constant $\epsilon > 0$, independent of snr, and all $m \in \{1, \cdots, M\}$, we have

$$\begin{aligned}
\mathbb{P}\left\{\mathcal{I}_{\pi_m,m} \leq \frac{\epsilon}{\mathsf{snr}}\right\} &\geq \mathbb{P}\left\{\max_{1\leq m\leq M}\min_{j\in\mathcal{A}_m}\mathcal{I}_{j,m} \leq \frac{\epsilon}{\mathsf{snr}}\right\} \\
&\overset{(\mathrm{a})}{\geq} 1 - \mathbb{P}\left\{\exists m : \min_{j\in\mathcal{A}_m}\mathcal{I}_{j,m} \geq \frac{\epsilon}{\mathsf{snr}}\right\} \\
&\overset{(\mathrm{b})}{\geq} 1 - M\,\mathbb{P}\left\{\min_{j\in\mathcal{A}_m}\mathcal{I}_{j,m} \geq \frac{\epsilon}{\mathsf{snr}}\right\} \\
&\overset{(\mathrm{c})}{=} 1 - M\left(1 - F_{\mathcal{I}_{1,m}}\left(\frac{\epsilon}{\mathsf{snr}}\right)\right)^{|\mathcal{A}_m|} \\
&\overset{(\mathrm{d})}{\geq} 1 - M\left(1 - C\left(\frac{\epsilon}{\mathsf{snr}}\right)^{\beta}\right)^{N-2M+1}, \quad (9)
\end{aligned}$$

where $C = \frac{e^{-1}(2(M-1)M)^{-\beta}}{\Gamma(\beta+1)}$; (a) holds from the De Morgan's law; (b) follows from the union bound; (c) follows since $\mathcal{I}_{j,m}$ $\forall j \in \mathcal{A}_m$ are the independent and identically distributed (i.i.d.) random variables for a given m, owning to the fact that the channels are i.i.d. and the beamforming matrix is unitary; and

[5]Note that when $M = 1$ the term $(M-1)M$ in (7) is replaced by one.

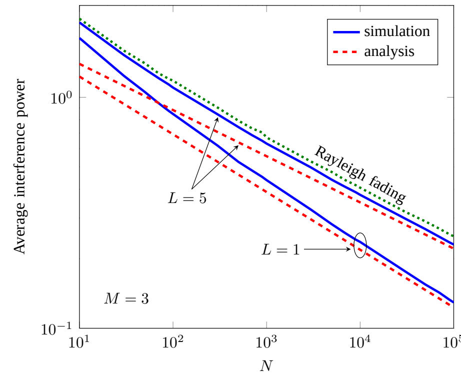

Fig. 1. The average interference power versus N for the FD system with $M = 3$. The following two cases are considered: i) $L = 1$ and ii) $L = 5$.

(d) follows from Lemma 1 since $0 < \frac{\epsilon}{\mathsf{snr}} \leq 2M(M-1)$ as $\mathsf{snr} \to \infty$ and from the fact that $|\mathcal{A}_m| \geq N - 2M + 1$. This completes the proof of the lemma. □

We are now ready to prove Theorem 1. From (5), if $\mathcal{I}_{\pi_m,m}\mathsf{snr} \leq \epsilon$ for $\epsilon > 0$ independent of snr for all $m \in \{1, \cdots, M\}$, then it follows from (3) and (6) that the sum DoF of M is achievable for downlink transmission. From Lemma 2, $\mathbb{P}\left\{\mathcal{I}_{\pi_m,m} \leq \frac{\epsilon}{\mathsf{snr}}\right\}$ is lower-bounded by (8), which converges to one as $\mathsf{snr} \to \infty$ if $N = \omega\left(\mathsf{snr}^{\beta}\right)$. Hence, the sum DoF of M is achievable for downlink transmission whp if $N = \omega\left(\mathsf{snr}^{\beta}\right)$. In conclusion, $\mathsf{DoF} = 2M$ is achievable whp if $N = \omega\left(\mathsf{snr}^{\beta}\right)$, which completes the proof of Theorem 1.

Remark 1: It is not difficult to show that for large N,

$$\mathbb{E}\left\{\max_{1\leq m\leq M}\mathcal{I}_{\pi_m,m}\right\} \leq \mathcal{O}\left(N^{-\frac{1}{\beta}}\right).$$

Then, from (9) and the Markov's inequality, it follows that

$$\begin{aligned}
1 - \mathbb{P}\left\{\mathcal{I}_{\pi_m,m} \leq \frac{\epsilon}{\mathsf{snr}}\right\} &\leq \frac{M\mathsf{snr}}{\epsilon}\mathbb{E}\left\{\max_{1\leq m\leq M}\mathcal{I}_{\pi_m,m}\right\} \\
&= \mathcal{O}\left(\frac{\mathsf{snr}}{N^{1/\beta}}\right),
\end{aligned}$$

which tends to zero if $N = \omega\left(\mathsf{snr}^{\beta}\right)$. Thus, we obtain the same scaling law as in Theorem 1. This implies that the faster interference decaying rate with respect to N, the smaller SNR exponent in the user scaling law.

Remark 2: For rich scattering environments, i.e., $L \to \infty$, the DoF of $2M$ is achievable if $N = \omega\left(\mathsf{snr}^{2M-1}\right)$. We also note that when $L < M$, the user scaling law required to achieve full DoF gets reduced to $N = \omega\left(\mathsf{snr}^{L+M}\right)$. This can be attributed from the fact that as L decreases, the probability that the interfering links suffer from a deep fade is increased, which in turn results in a reduction on the SNR exponent in the user scaling law condition.

IV. Numerical Evaluation and Discussions

In Fig. 1, the log-log plot of the average interference power versus N is shown for the FD system when $M = 3$ and $L = 1, 5$. It can be seen that the interference power tends to decrease linearly with N. In this figure, the dashed lines are also plotted from theoretical results in Remark 1 with a proper bias to check their slopes. We can see that the interference power decaying rates are consistent with

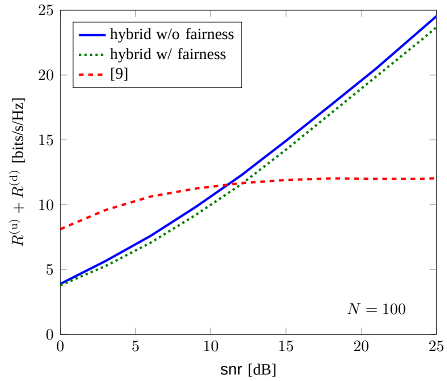

Fig. 2. The achievable sum-rates of the FD system by different user scheduling schemes when $M = 3$, $L = 5$, and $N = 100$.

the user scaling condition in Theorem 1. More specifically, the interference power is reduced as L decreases, but the slopes of the simulated curves remain almost the same when $L \geq M - 1$. This numerical result is sufficient to guarantee our DoF achievability in Section III.

To further ascertain the efficacy of our scheme, performance comparison is made with the existing scheme in [9]. The achievable sum-rates are illustrated in Fig. 2 according to snr when $L = 5$, $M = 3$, and $N = 100$. We can see that our scheme outperforms the conventional one beyond a certain low SNR point. This is because our scheme guarantees (at least) M DoF by taking advantage of the ZF receiver for uplink, thus resulting in infinitely large sum-rates with increasing snr. On the other hand, for fixed N, the sum-rates of the scheme in [9] are slightly changed as snr increases due to the residual interference at each dimension. In Fig. 2, the sum-rate curve of another scheme employing the proportional fair (PF) scheduler [10] with random beamforming for downlink transmission is also plotted, where the past window length is set to 100 time slots, which is a typical value to guarantee the fairness. As in the original hybrid scheme, the scheme applies the round-robin scheduler with ZF beamforming for uplink. We can see that the hybrid scheme with PF scheduling has a comparable sum-rate performance for practical N.

Appendix A
Proof of Lemma 1

From (1) and (2), the interference power defined in (4) can be expressed as

$$\mathcal{I}_{j,\ell} = \frac{1}{L}\boldsymbol{\alpha}_j^{\dagger}\boldsymbol{E}_j^{\dagger}\boldsymbol{\Phi}_{\ell}\boldsymbol{\Phi}_{\ell}^{\dagger}\boldsymbol{E}_j\boldsymbol{\alpha}_j + \frac{1}{L}\sum_{m=1}^{M}\left|\boldsymbol{\psi}_{j,m}\boldsymbol{\beta}_{j,m}\right|^2, \quad (10)$$

where $\boldsymbol{\Phi}_{\ell} = [\boldsymbol{v}_1 \ldots \boldsymbol{v}_{\ell-1}\, \boldsymbol{v}_{\ell+1} \cdots \boldsymbol{v}_M]$. Let $\boldsymbol{\Sigma} = \boldsymbol{E}_j^{\dagger}\boldsymbol{\Phi}_{\ell}\boldsymbol{\Phi}_{\ell}^{\dagger}\boldsymbol{E}_j$, $\boldsymbol{\Psi}_m = \boldsymbol{\psi}_{j,m}^{\dagger}\boldsymbol{\psi}_{j,m}$, and $(\boldsymbol{\Sigma})_{i,i}$ be the i-th diagonal element of the matrix $\boldsymbol{\Sigma}$. Then, we have

$$(\boldsymbol{\Sigma})_{i,i} = \sum_{m=1, m\neq j}^{M}\left|\boldsymbol{v}_m^{\dagger}\boldsymbol{e}_j^i\right|^2 \leq (M-1)\,M,$$

which comes from the Cauchy's inequality and the fact that $\boldsymbol{v}_m$'s are orthogonal vectors. Thus, the maximum eigenvalue of $\boldsymbol{\Sigma}$, $\lambda_{\max}(\boldsymbol{\Sigma})$, satisfies $\lambda_{\max}(\boldsymbol{\Sigma}) \leq (M-1)\,ML$. We also note that $\operatorname{rank}(\boldsymbol{\Sigma}) = \min(M-1, L)$, $\lambda_{\max}(\boldsymbol{\Psi}_m) = L$, and $\operatorname{rank}(\boldsymbol{\Psi}_m) = 1$. Let $\boldsymbol{\Sigma} = \boldsymbol{U}\boldsymbol{\Gamma}\boldsymbol{U}^{\dagger}$ be the eigenvalue decomposition of $\boldsymbol{\Sigma}$, where $\boldsymbol{U}$ is a unitary matrix whose columns contain eigenvectors of $\boldsymbol{\Sigma}$ and $\boldsymbol{\Gamma}$ is the diagonal matrix containing eigenvalues of $\boldsymbol{\Sigma}$. By combining all the above observations, for $M > 1$, the interference power can be upper-bounded by

$$\mathcal{I}_{j,\ell} \leq \boldsymbol{\alpha}_j^{\dagger}\boldsymbol{U}\grave{\boldsymbol{\Gamma}}\boldsymbol{U}^{\dagger}\boldsymbol{\alpha}_j + \left(M^2 - M\right)\sum_{m=1}^{M}\boldsymbol{\beta}_{j,m}^{\dagger}\begin{pmatrix}1 & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{pmatrix}\boldsymbol{\beta}_{j,m}, \quad (11)$$

where the inequality holds since $\frac{1}{L}\boldsymbol{\Sigma} \preceq \boldsymbol{U}\grave{\boldsymbol{\Gamma}}\boldsymbol{U}^{\dagger}$ and $M > 1$ with[6]

$$\grave{\boldsymbol{\Gamma}} = \begin{cases} (M-1)\,M\mathbf{I}_L & L < M-1, \\ \begin{pmatrix} (M-1)\,M\mathbf{I}_{M-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{pmatrix} & \text{otherwise.}\end{cases}$$

Due to the unitary invariance of Gaussian distribution, the right-hand side of (11) can be expressed as $(M-1)\,M\chi^2(2\beta)$, where $\chi^2(k)$ denotes the Chi-squared random variable with k degrees of freedom. Therefore, it follows that

$$\begin{aligned} F_{\mathcal{I}_{j,\ell}}(\xi) &\geq \mathbb{P}\left\{\chi^2(2\beta) \leq \frac{\xi}{(M-1)\,M}\right\} \\ &\geq \frac{\gamma\left(\beta, \xi/\left(2\,(M-1)\,M\right)\right)}{\Gamma(\beta)} \\ &\geq \frac{e^{-1}\left(2\,(M-1)\,M\right)^{-\beta}}{\Gamma(\beta+1)}\xi^{\beta}, \end{aligned}$$

where $\Gamma(\cdot)$ and $\gamma(\cdot,\cdot)$ are the Euler gamma and incomplete gamma functions, respectively , and the last inequality follows from [5], which results in (7).

References

[1] E. Aryafar, M. Khojastepour, K. Sundaresan, S. Rangarajan, and M. Chiang, "MIDU: Enabling MIMO full-duplex," in *Proc. ACM MobiCom'12*, Istanbul, Turkey, Aug. 2012, pp. 257–268.

[2] J. Bai and A. Sabharwal, "Distributed full-duplex via wireless side channels: Bounds and protocols," *IEEE Trans. Wireless Commun.*, vol. 12, no. 8, pp. 4162–4173, July 2013.

[3] A. Sahai, S. Diggavi, and A. Sabharwal, "On degrees-of-freedom of full-duplex uplink/downlink channel," in *Proc. ITW'13*, Sevilla, Spain, Sept. 2013, pp. 1–5.

[4] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channels with partial side information," *IEEE Trans. Inf. Theory*, vol. 51, no. 2, pp. 506–522, Feb. 2005.

[5] B. C. Jung, D. Park, and W.-Y. Shin, "Opportunistic interference mitigation achieves optimal degrees-of-freedom in wireless multi-cell uplink networks," *IEEE Trans. Commun.*, vol. 60, no. 7, pp. 1935–1944, July 2012.

[6] A. G. Burr, "Capacity bounds and estimates for the finite scatterers MIMO wireless channel," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 5, pp. 812–818, June 2003.

[7] M. Akdeniz, Y. Liu, M. Samimi, S. Sun, S. Rangan, T. Rappaport, and E. Erkip, "Millimeter wave channel modeling and cellular capacity evaluation," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1164–1179, June 2014.

[8] A. Alkhateeb, O. E. Ayach, G. Leus, and R. W. Heath Jr., "Channel estimation and hybrid precoding for millimeter wave cellular systems," *IEEE J. Sel. Topics Sig. Process.*, vol. 8, no. 5, pp. 831–846, Oct. 2014.

[9] C. Karakus and S. N. Diggavi, "Opportunistic scheduling for full-duplex uplink-downlink networks," preprint. [Online]. Available: http://arxiv.org/abs/1504.05898.

[10] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277–1294, June 2002.

[6] Löwner partial ordering for Hermitian matrices $\mathbf{A}$ and $\mathbf{B}$: $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is positive semidefinite.